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APPROXIMATE TECHNIQUE FOR ESTIMATING LABYRINTH ATTENUATION OF  
ACCELERATOR-PRODUCED NEUTRONS

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## INTRODUCTION

In the design of accelerator radiological shielding, practitioners at Fermilab and elsewhere generally follow an established methodology in which the problem is factored into a source term and the attenuation factors for the various "legs" of the maze. The general methodology has been discussed in the literature and reviewed in detail by Cossairt<sup>1</sup>. The purpose of the present note is to present a methodology for simplifying such calculations in a form that makes them more amenable to the employment of spreadsheets. Figure 1 shows a typical three-legged labyrinth and defines the coordinates used throughout this paper.

## METHODOLOGY-SOURCE TERM

One can, in general, calculate the so-called source term, typically the dose equivalent or neutron flux density found at the entrance from the beam enclosure into the so-called "first leg" of the labyrinth, by employing Monte-Carlo techniques. However, in practical situations this has found not to be necessary. For high energy proton accelerators, a rule of thumb for the source term which has been found to be very successful for the degree of accuracy generally required for personnel protection purposes has been developed at Fermilab by Ruffin and Moore<sup>2</sup>, and more recently improved by inclusion of the Moyer Model energy scaling by Rameika<sup>3</sup>. In this model, it is observed that about one fast neutron/GeV of proton beam energy is produced with an isotropic distribution, in addition to the much higher multiplicity in the forward direction. (This may have a simple-minded explanation, or at least mnemonic device, in that 1 GeV is the approximate mass of the nucleon!) The neutrons which will dominate the spectrum and determine the dose equivalent at the entrance to the labyrinth are those around 1 to 10 MeV of kinetic energy. From the fluence to dose equivalent information, 8.3 neutrons/(cm<sup>2</sup> s) corresponds to a dose equivalent rate of 1 mrem/hour. Hence,  $3 \times 10^7$  neutrons/cm<sup>2</sup> approximately corresponds to 1 rem.

Thus, at distance R from the source, Rameika obtains

$$H(\text{rem}) = \frac{E_p^{0.8} N_p}{4\pi R^2 (3 \times 10^7)} = \left( 2.65 \times 10^{-9} \right) \frac{E_p^{0.8} N_p}{R^2} . \quad (1)$$

where R is in cm,  $E_p$  is in GeV, and  $N_p$  is the number of incident hadrons, typically protons for primary beams. The constant,  $2.65 \times 10^{-9}$  (rem cm<sup>2</sup>), turns out to be approximately 1/3 the value obtained by using the Moyer source parameter (see Chapter 3 of Ref. 1) along with the Moyer angular factor at  $\theta = \pi/2$ ;

$$(2.8 \times 10^{-7} \text{ rem cm}^2) \exp(-2.3\pi/2) = 7.6 \times 10^{-9} \text{ rem cm}^2. \quad (2)$$

It is, in fact expected that the value of this constant for a labyrinth would be less than the Moyer source term due to the fact that the Moyer source term implicitly assumes the development of the shower in the enclosure shielding which is, in effect, "short-circuited" by the passageway. Thus, one can, knowing the proton energy ( $E_p$ ), intensity ( $N_p$ ), and the distance from the source, (generally the beamline) to the "mouth" of the labyrinth (R), readily compute the dose at the "mouth" of the maze.

## METHODOLOGY-ATTENUATION CURVES

### First Leg Attenuation

K. Goebel has made a detailed comparison of various Monte-Carlo calculations of labyrinth attenuation<sup>4</sup>. In particular, he has calculated "universal" curves for "first" legs of labyrinths, i.e., the first leg proceeding outward from the beam enclosure. Goebel used and compared results from the codes SAM-CE<sup>5</sup> (Co73), AMC<sup>6</sup>, and ZEUS<sup>7</sup>. Gollon and Awschalom<sup>8</sup> have generated similar curves using the ZEUS code for a variety of geometries. The three conditions of point source, line source, and plane source or point-source off-axis for a straight tunnel displayed as universal dose attenuation curves as a function of the distance into the tunnel in units of the square root of the cross-sectional area as calculated by Goebel are given in Fig. 4.6 of Ref. 1 as taken from Ref. 4. In practical usage, "point" source condition is the one most generally used for radiation protection purposes. K. Tesch<sup>9</sup> has observed that the dependence of the dose equivalent or flux density with distance in such a "first leg" is dominated by a dependence that is approximately "inverse-square". Taking that as a starting point, the point source dependence calculated by Goebel can be approximated by the following expression:

$$H_1(d) = \left[ \frac{r_o}{d_1 + r_o} \right]^2 H_o \quad (3)$$

where  $r_o$  is simply a fitting parameter that has the value of

$$r_o = 1.4 \quad (4)$$

and where  $d_1$  is the distance measured from the mouth of the passageway in "units" of the square root of the cross-sectional area of the first leg.  $H_o$  is the dose at the mouth determined, for example, from Eq (1).

The quality of this fit is displayed in Figure 2.\* Over the domain of  $0 < d_1 < 9$  the expression fits the Goebel curve within  $\pm 10$  per cent, certainly sufficiently accurate for radiation protection purposes. The domain of  $d_1$  is the appropriate one given the fact that most "personnel" labyrinths are of cross-sectional area of about  $1 \times 2 \text{ m}^2$ . Hence, the unit length is approximately 1.4 meters. A 10 "unit" long first leg is, typically, 14 meters (or about 46 feet) which is quite long compared with normally-encountered labyrinth legs.

### Second and Succeeding Leg Attenuation

For rectilinear labyrinths, the thought is that the attenuation in all legs after the first is identical. This has been shown to be approximately true in actual measurements (see Ref. 10). Goebel, in Ref. 4, also examined this situation. He identified a "preferred" attenuation curve for such legs which appears to give results that compare well with measurements, as reported by Thomas and Stevenson<sup>11</sup>. Previously, Tesch<sup>9</sup> had found that it was possible to account for attenuation of the legs of a labyrinth beyond the first by employing the sum of two exponentials as follows:

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\* In both Figs. 2 and 3, Goebel's curves appear to be a bit "rough". This is due to the fact that they were obtained by measuring plotted curves rather than from numerical tabulations which were unavailable to the author.

$$H_i(r_i) = \left\{ \frac{\exp(-r_i/0.45) + 0.022A_i^{1.3} \exp(-r_i/2.35)}{1 + 0.022A_i^{1.3}} \right\} H_{i-1} \quad i^{\text{th}} \text{ leg } (i > 1) \quad (5)$$

where, for the  $i^{\text{th}}$  leg,  $H_{i-1}$  is the dose equivalent at the entrance to it,  $r_i$  is the distance into it (in meters), and  $A_i$  is the cross sectional area of the enclosure (meters<sup>2</sup>). Thus Eq. (5) is used "recursively" to determine the dose equivalent at the exit of each leg. This formula is easily used on a small calculator, but does not contain the expected scaling with the square root of the tunnel aperture. It is best used for personnel (i.e., "door-sized") labyrinths having cross sectional areas of approximately 2 m<sup>2</sup>.

In the present work, perhaps inspired by that of Ref. 9, it is found that the "2<sup>nd</sup> Leg Attenuation" factor "preferred" by Goebel can be quite adequately fit by the sum of three exponentials in the form of the following equation,

$$H_i(d_i) = \left\{ \frac{\exp(-d_i/r_1) + K \exp(-d_i/r_2) + J \exp(-d_i/r_3)}{1 + K + J} \right\} H_{i-1} \quad i^{\text{th}} \text{ leg } (i > 1) . \quad (6)$$

In Eq. 6,  $d_i$  is the distance from the entrance (working out from the beam) to the point where the dose equivalent is desired in the  $i^{\text{th}}$  leg in units of the square root of the cross-sectional area of the labyrinth. The fitting parameters,  $r_1$ ,  $r_2$ ,  $r_3$ ,  $K$ , and  $J$ , given below were found to result in an approximation to the Goebel curve that, over the domain of  $0 < d_i < 9$ , is within  $\pm 10$  per cent. As was the situation for the first leg, this domain of "good fit" includes legs of "reasonable" length.

$$r_1 = 0.17$$

$$r_2 = 1.17$$

$$r_3 = 5.25$$

$$K = 0.21$$

$$J = 0.00147$$

The agreement is illustrated in Figure 3 which shows both the Goebel "preferred" curve and the plot of Eq. (6) using the specified parameter set.

## SPREADSHEET FOR LABYRINTH CALCULATIONS

As a result of this exercise, a spreadsheet has been developed employing Microsoft Excel™ to perform such calculations. The spreadsheet uses the results of this work to calculate the dose equivalent after every stage of a labyrinth up to four. The spreadsheet is designed to be rather self-explanatory. Table 1 is a copy of this spreadsheet showing actual results while Table 2 shows the actual formulae in the spreadsheet. One should take care that the formulae in Table 2 make certain modifications of parameter names in order to provide for valid syntax for the spreadsheet. To use the spreadsheet, one simply needs to enter the specified quantities in the portion of the spreadsheet bordered by double lines. The units for each item are indicated on the spreadsheet. The results will be calculated by the spreadsheet and will appear in the boxes bordered by the heavy lines. At the top of the spreadsheet, one needs to enter the beam energy ( $E_p$ ), number of protons incident ( $N_p$ ), and distance from the source to the mouth of the labyrinth ( $R$ ). The source term at the mouth of the labyrinth is then calculated. Below that one enters the cross-sectional areas of the legs of the labyrinth followed by the lengths of each leg. The spreadsheet then proceeds to calculate the dose equivalent at the end of each leg as a result of the loss of the specified number of protons. Finally, the fitting parameters are displayed at the bottom of the spreadsheet. The example spreadsheet shown in Table 2 is a calculation involving  $1E13$  protons of 800 GeV energy incident on a loss point 3 meters from the entrance to a four-legged labyrinth. Each leg of the labyrinth has a cross-sectional area of  $3 \text{ m}^2$ . All legs are 6 meters long.

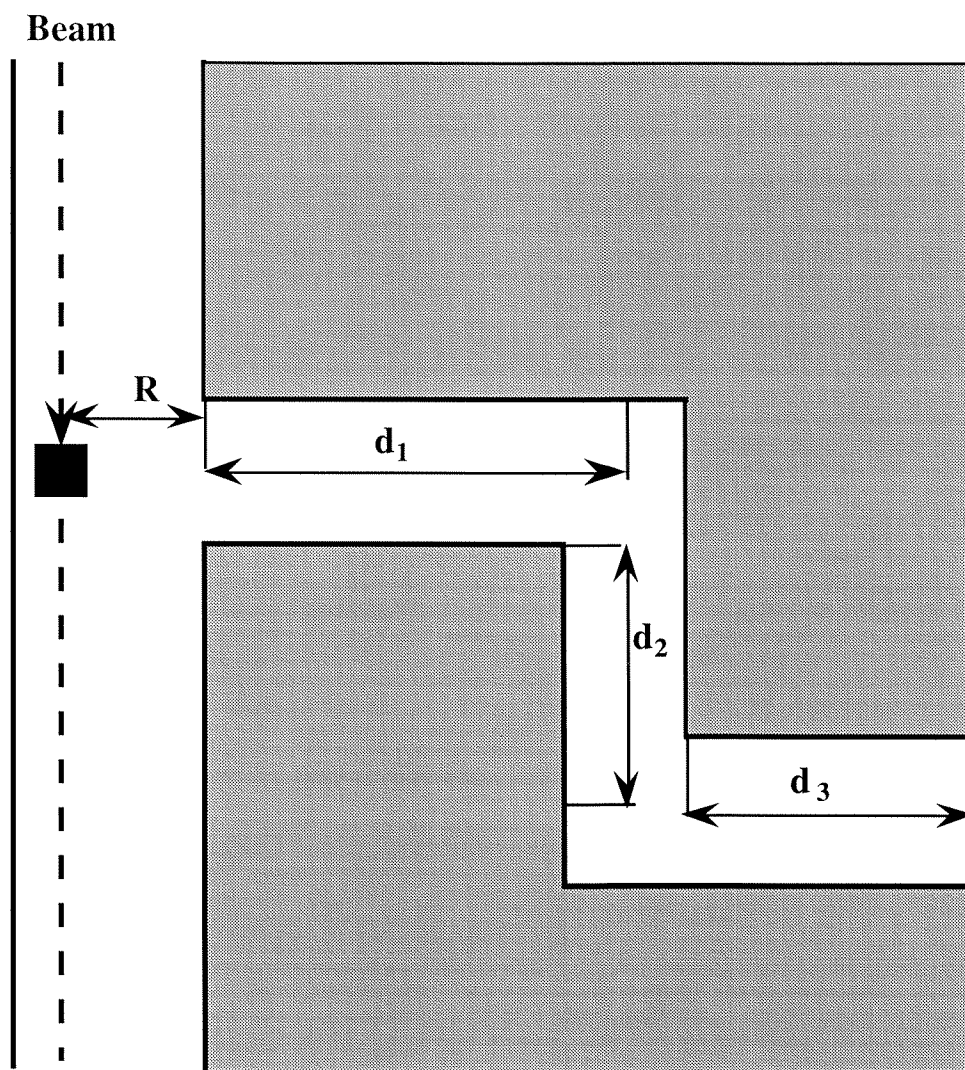


Figure 1 Typical Three-legged labyrinth showing coordinates used in the text. The cross-sectional area of each leg is determined by taking the square root of the width times the height.

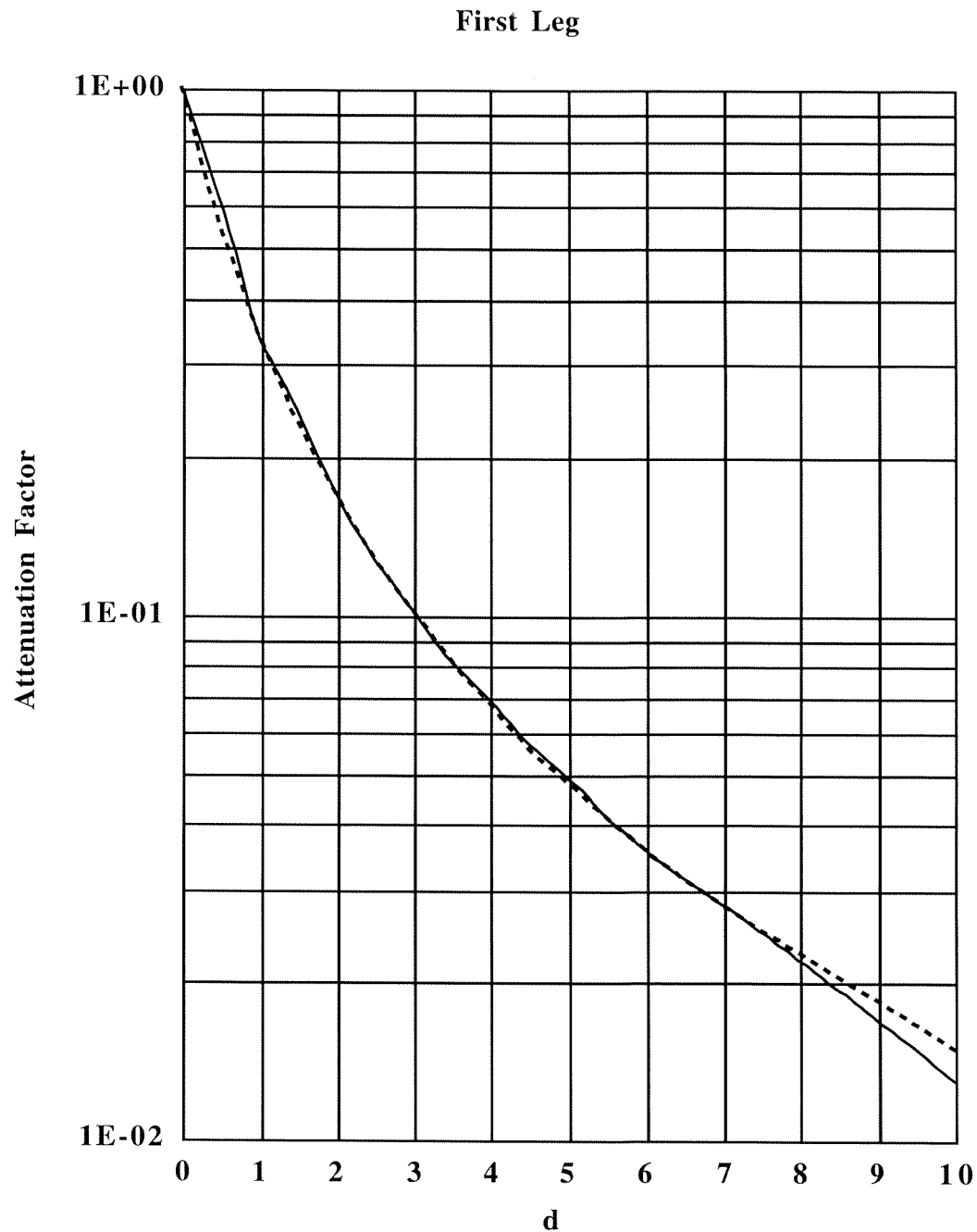


Figure 2 Plot of attenuation factor for the first leg. The solid curve is the attenuation factor recommended by Goebel in Ref. 4 while the dashed curve is that obtained by using Eq. (3). The quantity  $d$  is the center line distance along the first leg measured from the entrance to the penetration in units of the square root of its cross-sectional area. It is equivalent to the coordinate  $d_1$  as defined in Fig. 1.

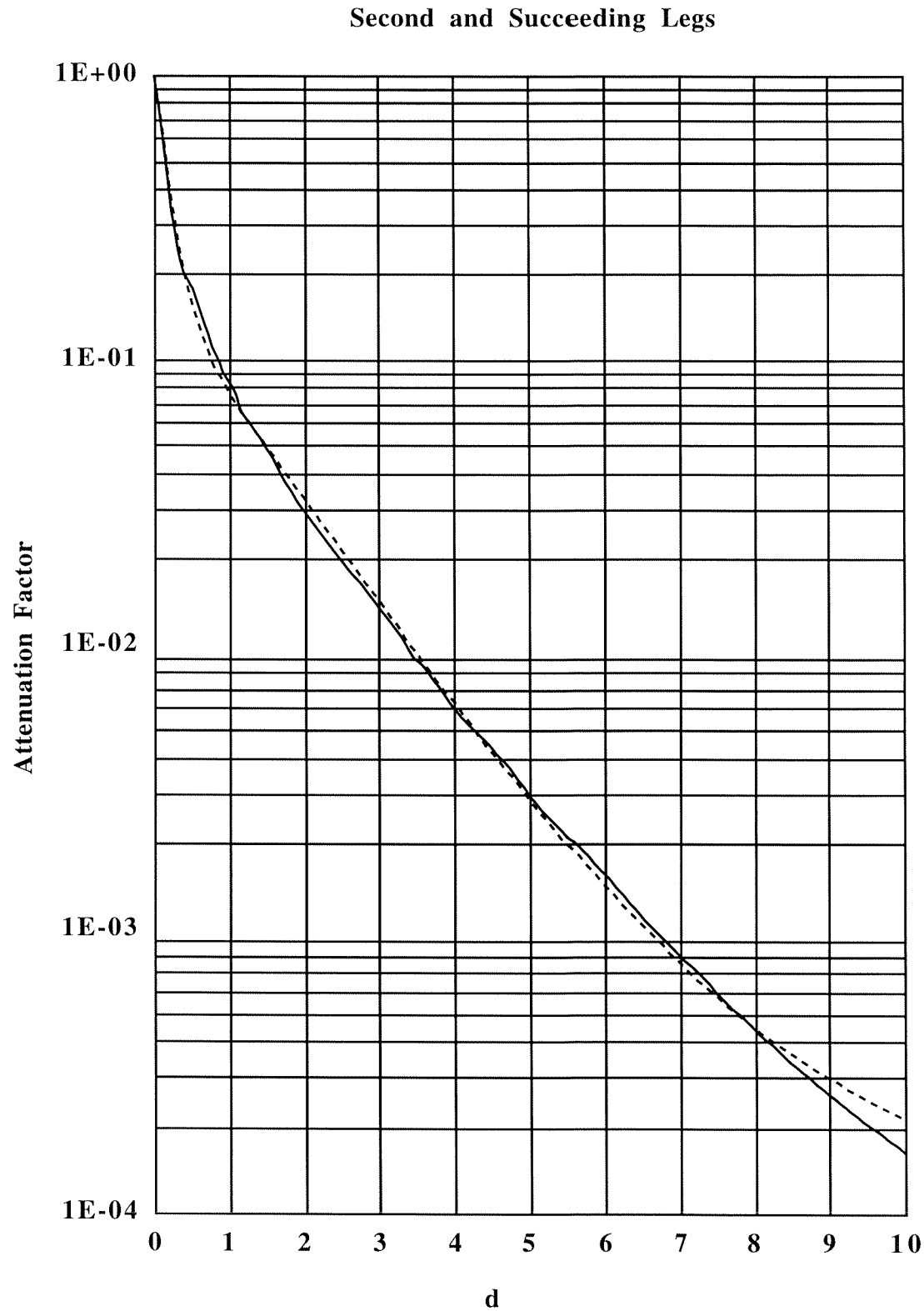


Figure 3 Plot of attenuation factor for second and succeeding legs. The solid curve is the attenuation factor recommended by Goebel in Ref. 4 while the dashed curve is that obtained by using the sum of three exponentials developed in the present work as Eq. (6). The quantity  $d$  is the center line distance measured from the entrance to the leg in units of the square root of its cross-sectional area. It is equivalent to the coordinate  $d_i$  (i .1) as defined in Fig. 1.



Table 1 Output of a Microsoft Excel™ spreadsheet employing formulae developed in this work.

A 1	B	C	D
2	Ep=	8 0 0	GeV
3	Np=	1.00E+13	Protons
4	R=	3	meters
5			
6	Source Term=	6.19E+04	mrem
7			
8	Leg 1 area=	3	m ** 2
9	Leg 2 area=	3	m ** 2
10	Leg 3 area =	3	m ** 2
11	Leg 4 area =	3	m ** 2
12			
13	Leg 1 length=	6	meters
14	Leg 2 length=	6	meters
15	Leg 3 length=	6	meters
16	Leg 4 length=	6	meters
17			
18	Leg 1 ("units")=	3.46	
19	Leg 2 ("units")=	3.46	
20	Leg 3 ("units")=	3.46	
21	Leg 4 ("units")=	3.46	
22			
23		<b>Atten.Fact.</b>	
24	Leg 1	8.28E-02	
25	Leg 2	9.60E-03	
26	Leg 3	9.60E-03	
27	Leg 4	9.60E-03	
28			
29		<b>D.E. After Leg</b>	
30		<b>(mrem)</b>	
31	Leg 1	5.13E+03	
32	Leg 2	4.92E+01	
33	Leg 3	4.73E-01	
34	Leg 4	4.54E-03	
35			
36	<b>Parameters:</b>		
37	ro =	1.4	
38	r1 =	0.17	
39	r2 =	1.17	
40	r3 =	5.25	
41	K =	0.21	
42	J =	0.00147	

Table 2 Same Microsoft Excel™ spreadsheet as in Table 1 only showing formulae.

A 1	B	C	D
2	Ep=	800	GeV
3	Np=	100000000000000	Protons
4	R=	3	meters
5			
6	Source Term=	=0.00000000265*(Ep^0.8)*Np*1000/(_R*100)^2	mrem
7			
8	Leg 1 area=	3	m**2
9	Leg 2 area=	3	m**2
10	Leg 3 area =	3	m**2
11	Leg 4 area =	3	m**2
12			
13	Leg 1 length=	6	meters
14	Leg 2 length=	6	meters
15	Leg 3 length=	6	meters
16	Leg 4 length=	6	meters
17			
18	Leg 1 ("units")=	=C13/SQRT(C8)	
19	Leg 2 ("units")=	=C14/SQRT(C9)	
20	Leg 3 ("units")=	=C15/SQRT(C10)	
21	Leg 4 ("units")=	=C16/SQRT(C11)	
22			
23		Atten.Fact.	
24	Leg 1	=(ro/(C18+ro))^2	
25	Leg 2	=(EXP(-C19/_r1) + _K*EXP(-C19/_r2)+_J*EXP(-C19/_r3))/(1+_K+_J)	
26	Leg 3	=(EXP(-C20/_r1) + _K*EXP(-C20/_r2)+_J*EXP(-C20/_r3))/(1+_K+_J)	
27	Leg 4	=(EXP(-C21/_r1) + _K*EXP(-C21/_r2)+_J*EXP(-C21/_r3))/(1+_K+_J)	
28			
29		D.E. After Leg	
30		(mrem)	
31	Leg 1	=C6*C24	
32	Leg 2	=C25*C31	
33	Leg 3	=C26*C32	
34	Leg 4	=C27*C33	
35			
36	Parameters:		
37	ro=	1.4	
38	r1=	0.17	
39	r2=	1.17	
40	r3=	5.25	
41	K=	0.21	
42	J =	0.00147	

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